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Let $x =$ the sum of their rates during second hour.

In the 12 minutes after passing B the third time A gains 8/15 mile. Hence, after A reverses his direction, the three meetings will require them to travel $2\frac{8}{15}$ miles at x miles an hour, or $x/60$ mile per minute. Then $2\frac{8}{15} \div x/60 = 152/x$ is the number of minutes required for the three meetings.

After this they continue in opposite directions 6 minutes, and together travel $x/10$ miles.

Case I.—If they do not meet during the 6 minutes, then A, after resuming his original direction, must first gain $x/10$ miles to overtake B, which requires $x/10 \times 22.5$ minutes. Hence

$$12 + \frac{152}{x} + 6 + \frac{x}{10} \times 22.5 = 180 - (67.5 + 67.5) = 45 \text{ minutes.}$$

or,

$$\frac{9x}{4} + \frac{153}{x} = 27, \tag{1}$$

from which $x = 18 \pm \sqrt{-284}$.

Case II.—If they meet once during the 6 minutes, then A must gain $(x/10 - 1)$ miles, which he will do in $(x/10 - 1) \times 22.5$ minutes.

Then equation (1) becomes

$$\frac{9x}{4} - 22.5 + \frac{152}{x} = 27, \tag{2}$$

from which $x = 18.31$.

Hence A's rate is now 10.49 miles per hour and B's is 7.82 miles per hour. Then A began walking at 11.49 miles per hour and B at 8.82 miles per hour. In this way a complete time-table has been constructed.

GEOMETRY.

407. Proposed by S. LEFSCHETZ, University of Nebraska.

To construct a right triangle knowing the sum of the sides of the right angle and the sum of one of them and the hypothenuse.

I. Solution by A. H. Holmes, Brunswick, Maine.

Let a be the sum of the two sides and b be the sum of one side and the hypothenuse of the required triangle. Draw $AB = 2a$ and extend to C, making $BC = b - a$. On AC describe the semi-circle, ACD. At B erect a perpendicular BD cutting the circumference in D. On DB take $DE = BC$. Then BE is one of the required sides. From O, the middle point of AB, lay off $OF = BE$. Then BF is the other side and BEF is the required triangle. For, letting x and y represent the sides, we have $x + y = a$ and $x + \sqrt{x^2 + y^2} = b$, from which $x = \sqrt{2b(b - a)} - (b - a)$ and $y = b - \sqrt{2b(b - a)}$.

II. SOLUTION BY T. M. BLAKSLEE, Ames, Iowa.

Given $b + a$ and $a + h$, b, a, and h being the base, altitude, and hypothenuse respectively. Draw two parallel rays, l and s at distance $a + h$. On l lay off $AD = b + a$. Draw DE making $\angle ADE = 45°$. The ray of DE contains the vertex B of the required triangle ACB. This vertex is also on the parabola having s as directrix and A as focus.

Having found B, draw BC perpendicular to l and join A and B.

Excellent solutions were also received from A. M. Harding, C. N. Schmall, H. C. Feemster, Levi S. Shively, Elmer Schuyler, W. T. Risley, and Francis C. Rust.

408. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Given a point A on a circle and a chord of the circle; to draw a chord through A so that it shall be bisected by the given chord.

SOLUTION BY A. M. HARDING, University of Arkansas.

Let C be the center of the circle. On CA as a diameter describe a circle. This circle will cut the chord, in general, in two points P_1, P_2. Join A to P_1 and A to P_2. Then these are the required chords. If A be on the minor arc of the circle there will be two solutions. If A be on the major arc there will be two solutions, one solution, or no solution according as d is less than, equal to, or greater than $r - c$, where d is the distance from A to the given chord, c is distance from center to the given chord and r is the radius of given circle.

Also solved by T. M. Blakslee, C. N. Schmall, A. H. Holmes, Levi S. Shively, Francis C. Rust, G. W. Hartwell, W. R. Lebold, H. C. Feemster, and Elmer Schuyler.

CALCULUS.

326. Proposed by C. N. SCHMALL, New York City.

Prove that

$$\int_0^\infty \frac{(\tan^{-1} ax)^2 - (\tan^{-1} bx)^2}{x}\, dx = \frac{\pi^2}{4}(\log a - \log b).$$

SOLUTION BY THE PROPOSER.

Take the integral,

$$u = \int_0^\infty [f(x) - f(kx)] \frac{dx}{x}. \tag{1}$$

Substitute for x the successive values kx, k^2x, k^3x, $\cdots$, k^nx.

We then have

$$u = \int_0^\infty [f(kx) - f(k^2 x)] \frac{dx}{x}, \tag{2}$$

$$u = \int_0^\infty [f(k^2 x) - f(k^3 x)] \frac{dx}{x}, \tag{3}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$u = \int_0^\infty [f(k^{n-1} x) - f(k^n x)] \frac{dx}{x}. \tag{n}$$